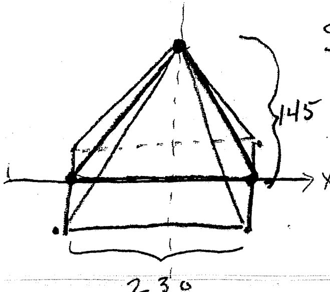
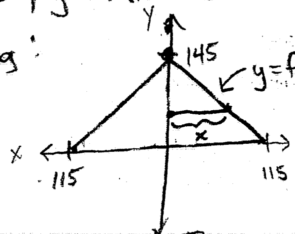
Classroom Activity: Volumes!

The Great Pyramid of Giza, completed in 2,560 BC was the tallest man-made structure for an estimated 3,800 years. At the time of construction (there has been significant erosion), it was approximately 145 meters tall and 230 meters wide at each base. The corners of the great pyramid point in the four cardinal directions: North, South, East, and West. The accuracy of the pyramid is astonishing: the four sides and base have an average error of only 58 millimeters.

A) ~~Estimate~~ Find the volume of the great pyramid to at least two significant digits. The pyramid does contain two interior chambers, but their relative volume is negligible.



Slice the pyramid vertically through the apex to get the drawing:



We wish to slice the pyramid horizontally giving squares whose side lengths are twice the 'x' in the diagram to the left.

The equation for the line on the right is:

$y = -\frac{145}{115}x + 145$ which is equivalent to: $x = -\frac{23}{29}(y-145)$

The area of each cross-sectional square is: $A(y) = 2x^2 = 4\left[-\frac{23}{29}(y-145)\right]^2$

$= 4\cdot\frac{23^2}{29^2}(y^2-290y+145^2)$

Thus, the volume of the pyramid is: $4\cdot\frac{23^2}{29^2}\int_0^{145} y^2-290y+145^2\,dy$

$= 4\cdot\frac{23^2}{29^2}\left(\frac{145^3}{3} - 290\frac{(145)^2}{2} + 145^3\right) = 2,556,833\,m$

B) There is not general agreement about how many blocks are used in the pyramid, (it's difficult to count any that aren't on the outside!). Say for the sake of our calculations that the average block is 1m x 2m x 3m in size. Approximately how many blocks are in the great pyramid?

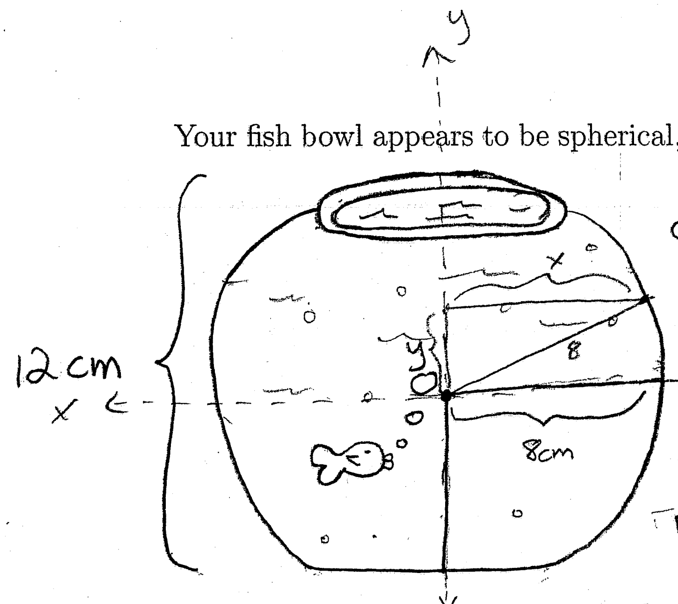
Each block is $6m^3$, so it would require

$\frac{2,556,833}{6} \approx 426,150$ blocks to fill the volume

C) It took 20 years to build the great pyramid. Approximately how many blocks had to be placed EACH DAY in order to complete this task?

$\frac{426,150 \text{ blocks}}{365 \times 20 \text{ days}} \approx 58.4 \text{ blocks/day}$

Your fish bowl appears to be spherical, but the top and bottom are symmetrically 'missing':



Each slice in the horizontal direction is a circle, so we desire an equation for the radii of these circles in terms of y.

The bowl is defined by rotating the region $x=0$, $x^2+y^2=64$, $y=-6$, $y=6$ around the y-axis. Thus $x=\sqrt{64-y^2}$ is the radius of each cross-section.

Volume of Fishbowl $= \int_{-6}^{6} \pi(64-y^2)\,dy = 2\pi\int_0^6 64-y^2\,dy$

$= 2\pi\left(64(6)-\frac{6^3}{3}\right)$

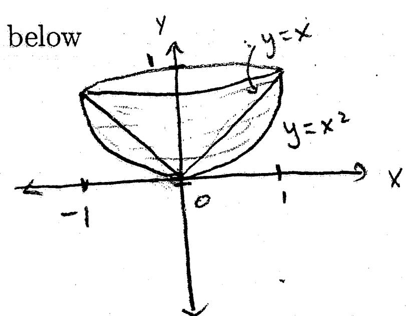
$= 624\pi \text{ cm}^3$

After removing your fish to a safe place and cleaning out his bowl, you put it under the sink faucet to refill it. If the water is coming out of the faucet at a rate of 30 cm^3/sec, how long will it take to refill your fishbowl?

$\frac{624\pi \text{ cm}^3}{30 \frac{\text{cm}^3}{\text{sec}}} \approx 65.3$ seconds.

The region enclosed by the curves $y=x$, $y=x^2$ is rotated around the y-axis to create a solid.

A) Sketch this solid below



B) Find the volume of this solid using the washer method:

The washers are horizontal, thus we must integrate w.r.t. y.

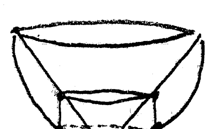
Outer area : $\pi(\sqrt{y})^2$

Inner area : $\pi(y)^2$

$V=\int_0^1 \pi(y-y^2)\,dy$

$=\pi\left[\frac{y^2}{2}-\frac{y^3}{3}\right]_0^1 = \pi\left(\frac{1}{2}-\frac{1}{3}\right) = \pi\left(\frac{3}{6}-\frac{2}{6}\right) = \frac{\pi}{6}$ units^3

C) Find the volume of this solid using cylindrical shells:



Area of the shell is: $2\pi r\cdot h\cdot dx = 2\pi(x)(x-x^2)dx$

$V=\int_0^1 2\pi(x)(x-x^2)dx$

$=2\pi\int_0^1 (x^2-x^3)dx$

$=2\pi\left(\frac{x^3}{3}-\frac{x^4}{4}\right)_0^1 = 2\pi\left(\frac{1}{3}-\frac{1}{4}\right) = 2\pi\left(\frac{4}{12}-\frac{3}{12}\right) = 2\pi\left(\frac{1}{12}\right) = \frac{\pi}{6}$ units^3

D) If this solid were a bowl, how much volume would it hold?

Use the disc method: (discs of radius $y=x$)

$\int_0^1 \pi y^2\,dy = \pi\left[\frac{y^3}{3}\right]_0^1 = \frac{\pi}{3}$ units^3